A distance friction minimization-goals achievement model in data envelopment analysis
An application to efficiency improvement in local government finance in Japan

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Abstract

Data Envelopment Analysis (DEA) has become an established approach in the analysis of efficiency problems in both public and private sectors. The aim of this paper is to present and apply a newly developed, adjusted DEA model – emerging from a blend of a Distance Friction Minimization (DFM) and a Goals Achievement (GA) approach on the basis of the Charnes-Cooper-Rhodes (CCR) method – in order to generate a more appropriate efficiency-improving projection model in conventional DEA.

Our DFM model is based on a generalized distance friction function and serves to assist a Decision Making Unit (DMU) in improving its performance by a proper movement towards the efficiency frontier surface. Standard DEA models use a uniform input reduction or a uniform output augmentation in the improvement projections, but our DFM approach aims to generate a new contribution to efficiency enhancement strategies by deploying a weighted projection function, while it may address both input reduction and output augmentation as a strategy of a DMU. A suitable form of multidimensional projection functions mapping out efficiency improvement is given by a Multiple Objective Quadratic Programming (MOQP) model in conformity with a Euclidean distance.

Another novelty of our approach is the introduction of prior goals set by a DMU by using a GA approach. The GA model specifies a goal value for efficiency improvement in a DFM model. The GA model can compute the input reduction value or the output augmentation value in order to achieve in an optimal way a pre-specified goal value for the efficiency improvement. Using next the integrated DFM-GA model, we are able to develop an operational efficiency-improving projection that provides a clear orientation for actions of a DMU.

The above-mentioned DFM-GA model will be empirically illustrated by using a data set on cities in Hokkaido prefecture in Japan, where the aim is to increase the efficiency of local government finance mechanisms in these cities, based on various input and output performance characteristics. In summary, this paper presents a practical policy instrument that may have a great added value for decision making and planning of both public and private actors.

Keywords: Distance Friction Minimization, Goals Achievement, Data Envelopment Analysis (DEA), Efficiency Improving Projection, Local Government Finance

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1. Introduction

Data Envelopment Analysis (DEA) has become an established approach in the analysis of efficiency problems in both public and private sectors. A large number of studies show that efficiency analysis is an important but difficult topic. DEA was developed to analyze the relative efficiency of ‘Decision Making Units’ (DMUs) by constructing a piecewise linear production frontier, and projecting each agent (DMU) onto the frontier. A DMU that is located on the frontier is efficient, while a DMU that is not on the frontier is inefficient. An inefficient DMU can become efficient by reducing its inputs (or increasing its outputs). In the standard DEA-approach, this is achieved by a uniform reduction in all inputs (or uniform augmentation in all outputs). But in principle, there is an infinite number of improvements to reach the efficient frontier, so that there are many solutions for a DMU to enhance efficiency.

The existence of an infinite number of solutions to reach the efficient frontier has led to a stream of literature on the integration of DEA and Multiple Objective Linear Programming (MOLP), which was initiated by Golany (1988). In short, this literature proposes trajectories to efficiency by taking into account the preferences of the decision maker (DMU). Thus, the challenge is now to develop a methodology for projecting DMUs on the efficient frontier that does not include subjective valuations.

Suzuki et al. (2007a, 2007b, 2007c) proposed a Distance Friction Minimization (DFM) model in a DEA model that is based on a generalized distance friction function and serves to assist a DMU in improving its performance by a proper movement towards the efficiency frontier surface. Our DFM approach aims to generate a new contribution to efficiency enhancement strategies by deploying a weighted projection function, while it may address both input reduction and output augmentation as a strategy of a DMU. A suitable form of multidimensional projection functions mapping out efficiency improvement is given by a Multiple Objective Quadratic Programming (MOQP) model in conformity with a Euclidean distance.

A general efficiency improving projection model in combination with our DFM model is able to calculate either an input reduction value or an output augmentation value to reach an efficient score 1.000, although in reality this may be hard to achieve.

The aim of this paper is to present and apply a newly developed, adjusted DEA model – emerging from a blend of a Distance Friction Minimization (DFM) and a Goals Achievement (GA) approach on the basis of the Charnes-Cooper-Rhodes (CCR) method – in order to generate a more appropriate efficiency-improving projection model in conventional DEA. The GA model specifies a Goal Improvement Rate (GIR) of the total efficiency gap in the framework of a DFM model. The GA model can compute an input reduction value or an output augmentation value in order to achieve in an optimal way a prior goal value for the efficiency improvement.

The above-mentioned CCR-DFM-GA model will be empirically illustrated by using a data set on cities in Hokkaido prefecture in Japan, where the aim is to increase the efficiency of local government finance, based on various input and output performance characteristics of these cities. The relevance of our approach can be illustrated by referring to recent public financial deficits in Yubari city in Hokkaido prefecture, which was close to a financial bankruptcy in March 2007. Especially, the White Paper on local public finance (2007) illustrated clearly that the issue of the public financial deficits of cities and prefectures is an urgent concern in Japan. This paper proposes thus a policy instrument that may have a great added value for decision making and planning of public finance actors.

The paper is organized as follows. Section 2 discusses DEA and efficiency improvement projection methods. Next, Section 3 introduces our DFM methodology, while Section 4 proposes the new model which is a GA model in the framework of a DFM model. Section 5 presents then an application of the methodology to a comparative study of local government finance efficiency analysis in Japan. Finally, Section 6 offers some conclusions.
2. Efficiency Improvement Projection in DEA

The original formulation for DEA was given by Farrell (1957) who aimed to develop a measure for production efficiency. This work was elaborated by Charnes et al. (1978) who presented a quantitative measure for assessing the relative efficiency of DMUs in case of a frontier method that aims to determine the maximum volume of outputs, given a set of inputs. In this framework, it is possible to assess ex post the (in)efficiency of a production system using the distance to the production frontier (without any explicit assumptions on the production technology at hand). This is usually a deterministic analysis, which has a close resemblance to non-parametric linear programming. DEA has ever since become an operational tool for analyzing efficiency problems in both the private and the public sector, where (in)efficiency is interpreted as the relative distance from an actual situation to the optimal production frontier function.

DEA has been fully developed by Charnes et al. (1978) and later on by Banker et al. (1984) to analyze the efficient operation of DMUs as well as to determine improvements of inefficiency by means of a proper projection choice of a DMU, based on the ratio of the weighted sum of outputs to the weighted sum of inputs, given the requirement that these ratios are less than (or equal to) 1 for each DMU under consideration. The main goal is to determine in numerical terms the weights associated with each DMU in such a way that it may maximize the improvement of its efficiency.

The Charnes et al. (1978) model (abbreviated hereafter as CCR-input model) for a given DMU $j$ to be evaluated on any trial generally designated as DMU $o$ (where $o$ ranges over 1, 2, …, $J$) may then be represented as the following fractional programming ($FP_o$) problem:

$$
\begin{align*}
\max_{\theta} \quad & \theta \quad = \quad \frac{\sum_m u_m y_{so}}{\sum_s y_{so}} \\
\text{s.t.} \quad & \frac{\sum_m u_m y_{sj}}{\sum_s y_{sj}} \leq 1 \quad (j = 1, \ldots, J) \tag{2.1}
\end{align*}
$$

where $\theta$ is an objective variable (efficiency score), $y_{mj}$ is the volume of input $m$ ($m=1, \ldots, M$) for DMU $j$ ($j=1, \ldots, J$), and $y_{sj}$ the output $s$ ($s=1, \ldots, S$) of DMU $j$, while $u_m$ and $u_s$ are the weights given to input $m$ and output $s$, respectively.

Model (2.1) is often called an input-oriented CCR model, while its reciprocal (i.e., an interchange of the numerator and denominator in objective function (2.1), with a specification as a minimization problem under a proper adjustment of the constraints) is usually coined an output-oriented CCR model. Model (2.1) is obviously a fractional programming model, which may be solved stepwise by first assigning an arbitrary value to the denominator in (2.1), and by maximizing next the numerator. But it is preferable to transform (2.1) into a linear programming model, as shown below.

The CCR model (2.1) can be shown to have the following equivalent linear programming ($LP_o$) specification for any DMU $j$:

$$
\begin{align*}
\max_{\theta} \quad & \theta \quad = \quad \sum_s u_s y_{so} \\
\text{s.t.} \quad & \sum_m v_m x_{mo} = 1 \
\end{align*}
$$

(2.2)
\[-\sum_{m} v_{m}x_{mj} + \sum_{s} u_{s}y_{sj} \leq 0\]
\[v_{m} \geq 0, \quad u_{s} \geq 0\]

The dual problem of (2.2), \(DLP_{\omega}\), can be expressed by means of a real variable \(\theta\) using the following vector notation:

\[(DLP_{\omega})\]

\[
\begin{align*}
\text{min} & \quad \theta \\
\text{s.t.} & \quad \theta x_{\omega} - X\lambda \geq 0 \\
& \quad Y\lambda \geq y_{\omega} \\
& \quad \lambda \geq 0
\end{align*}
\] (2.3)

where the transposed (T) presentation \(\lambda = (\lambda_{1}, \cdots, \lambda_{j})^{T}\) is a non-negative vector (corresponding to the presence of slacks for each DMU), \(X\) an \((M \times J)\) input matrix and \(Y\) an \((S \times J)\) input matrix.

We can now define the input excesses \(s^{-} \in \mathbb{R}^{m}\) and the output shortfalls \(s^{+} \in \mathbb{R}^{s}\), and identify them as ‘slack’ vectors as follows:

\[s^{-} = \theta x_{\omega} - X\lambda \] (2.4)
\[s^{+} = Y\lambda - y_{\omega} \] (2.5)

Then we can solve the following two-stage LP problem in a straightforward way:

1. Solve \(DLP_{\omega}\). Let the optimal objective value be \(\theta^{*}\).

2. Given the value of \(\theta^{*}\), solve the following LP model using \((\lambda, s^{-}, s^{+})\) as slack variables:

\[\begin{align*}
\text{max} & \quad \omega = es^{-} + es^{+} \\
\text{s.t.} & \quad s^{-} = \theta^{*}x_{\omega} - X\lambda \\
& \quad s^{+} = Y\lambda - y_{\omega} \\
& \quad \lambda \geq 0, \quad s^{-} \geq 0, \quad s^{+} \geq 0
\end{align*}\] (2.6)

(2.7)

(2.8)

(2.9)

where \(\omega\) is an objective variable, and \(e\) a unit vector. For any inefficient DMU\(_{\omega}\), we can now define the reference set \(E_{\omega}\) based on the max-slack solution as obtained in step 1 and 2, as follows:

\[E_{\omega} = \{j|\lambda_{j} > 0\} \quad (j \in \{1, \cdots, J\})\] (2.10)

where \(E_{\omega}\) is a reference set for any inefficient DMU\(_{\omega}\). An optimal solution can then be expressed as follows:
\[ \theta^* x_o = \sum_{j \in E_o} x_j \lambda_j^* + s^- \]  
\[ y'_o = \sum_{j \in E_o} y_j \lambda_j^* - s^+ \]  

The improvement projection \( (\hat{x}_o, \hat{y}_o) \) is now defined in (2.13) and (2.14) as:

\[ \hat{x}_o = \theta^* x_o - s^- \]  
\[ \hat{y}_o = y'_o + s^+ \]  

These equations suggest that the efficiency of \((x_o, y_o)\) for DMU\(_o\) can be improved, if the input values are reduced radially by the ratio \( \theta^* \) and the input excesses \( s^- \) are eliminated (see Figure 1). Similarly, the efficiency can be improved, if the output values are augmented by the output shortfall \( s^+ \).

The original DEA models presented in the literature have thus far only focused on a uniform input reduction or a uniform output augmentation in the improvement projections, as shown in Figure 1 (\( \theta^* = OC'/OC \)). But, in principle, there is an infinite number of improvement projections on the efficient frontier line. The improvement projection of the original DEA models is only one solution, based on a projection related to a uniform input reduction or a uniform output augmentation. If we adopt a different perspective, this will of course lead to an other projection.

In the past decade several attempts have been made to integrate DEA and MOLP models (see e.g., Belton 1992, Belton and Vickers 1993, and Doyle and Green 1993). Most of the research was inspired by the pioneering research of Golany (1988) who tried to find efficient solutions in order to map out the efficiency frontier, in an interactive way. Later on Kornbluth (1991) was able to show the similarity between DEA problems and fractional MOLP problems. This similarity holds for both input-oriented and output-oriented models.

Most contributions on the integration of DEA and MOLP models find their origin in the standard CCR model or in the Banker et al. (1984) (abbreviated as BCC) model which provide the foundations of DEA. All such models aim to find a proper projection for an efficiency improvement for each inefficient DMU, based on a radial projection in which the input volumes are reduced (or the output values are augmented) by a uniform ratio.

It is noteworthy that the existence of an infinite number of efficiency improvement solutions has in recent years prompted a rich literature on the methodological integration of the MOLP and DEA model. As mentioned, the first contribution was offered by Golany (1988) who proposed an interactive MOLP procedure which aimed at generating a set of efficient points for a DMU. This model allows a decision-maker to select the preferred set of output levels, given the input levels. This model was used as a support tool for the selection of effective and efficient points for a
decision-making agency. Next, Thanassoulis and Dyson (1992) developed adjusted models which can be used to estimate alternative input and output levels in order to render relatively inefficient DMUs more efficient. These models are able to incorporate preferences for a potential improvement of individual input and output levels. The resulting target levels reflect the user’s relative preference over alternative paths to efficiency. Joro et al. (1998) demonstrated the analytical similarity between a DEA model and a Reference Point Model in a MOLP formulation from a mathematical viewpoint. Additionally, the Reference Point Model provides suggestions which make it possible to freely search on the efficient frontier for good solutions or for the most preferred solution based on the decision-maker’s preference structure. More recently, Halme et al. (1999) developed a Value Efficiency Analysis (VEA), which included the decision-maker’s preference information in a DEA model. The foundation of VEA originates from the Reference Point Model in a MOLP context. Here the decision-maker identifies the Most Preferred Solution (MPS), so that each DMU can be evaluated by means of the assumed value function based on the MPS approach. A further development of this approach was made by Korhonen and Siljämäki (2002) who took care of several practical aspects related to the use of a VEA. In addition, Korhonen et al. (2003) developed a multiple objective approach which allows for changes in the time frame. And finally, Lins et al. (2004) proposed two multi-objective approaches that determine the basis for a posteriori preference incorporation. The first model is coined MORO (Multiple Objective Ratio Optimization), which optimizes the ratios between observed and target inputs (or outputs) of a DMU. The second model is MOTO (Multiple Objective Target Optimization), which directly optimizes the target values.

These approaches dealt with the challenge to identify a target or a direction to render relatively inefficient DMUs more efficient, based on the decision-maker’s preference information. The various approaches have suggested that the solution of an efficient improvement problem is not only a search for just one point. Especially, the Reference Point Model (see Joro et al. 1998) has many possibilities to generate a great variety of solutions to render inefficient DMUs more efficient. Clearly, one remark is in order here: these approaches have to incorporate the decision-maker’s preference information. In this regard, Angulo-Meza and Lins (2002) make the following observation:

“There are disadvantages in the methods that incorporate a priori information, concerning subjectivity:

• The value judgments, or a priori information can be wrong or biased, or the ideas may not be consistent with reality.
• There may be a lack of consensus among the experts or decision-makers, and this can slow down or adversely affect the study.

Indeed, one may want to preserve the DEA spirit in the sense of not including a priori information.” (p. 232).

Given these considerations, we propose in our study a new improvement projection model, coined the Distance Friction Minimization (DFM) approach, which does not need to incorporate a value judgment of a decision-maker. In this approach a generalized distance friction function will be presented to assist a DMU in improving its efficiency by a smart movement towards the efficiency frontier surface. The direction of this efficiency improvement depends on the input/output data characteristics of the DMU. Each of these characteristics may have a different weight for the DMU. To achieve an appropriate rise in efficiency, it is thus necessary to take into account the various most appropriate input/output weights of these characteristics. It is then possible to define the projection functions for the minimization of the distance friction, using a Euclidean distance in weighted spaces. We will use here a MOQP model.

3. The Distance Friction Minimization (DFM) Approach

As mentioned, the improvement solution in the original CCR-input model imposes that the input values are reduced radially by a uniform ratio $\theta^*$ ($\theta^* = OD'/OD$ in Figure 2). That is to say, the improvement solution for any arbitrary inefficient DMU is D’ in Figure 2 (in cases where the input space is a non-weighted (i.e., normal) x-space). The general specification of a CCR model was frequently based on a normal x- or y-space (non-weighted space) (see Figure 1), in contrast to Figures 2 and 3, which are based on weighted x- or y-spaces. Weighted spaces can be investigated
regarding the distance frictions in improvement projections for input and output variables in the following way (see Cooper et al. 2006).

The \((v^*, u^*)\) values obtained as an optimal solution for formula (2.2) result in a set of optimal weights for DMUo. Then the efficiency score can be evaluated by:

\[
\theta^* = \frac{\sum_{x} u^*_{x} y^*_{so}}{\sum_{m} v^*_{m} x^*_{mo}} \quad (3.1)
\]

The denominator may arbitrarily be set equal to 1, and hence:

\[
\theta^* = \sum_{x} u^*_{x} y^*_{so} \quad (3.2)
\]

As mentioned earlier, \((v^*, u^*)\) is the set of most favourable weights for DMUo in the sense of maximizing the ratio scale. \(v^*_m\) is the optimal weight for the input item \(m\) and its magnitude expresses how much in relative terms the item is contributing to efficiency. Similarly, \(u^*_s\) does the same for the output item \(s\). Furthermore, if we examine each item \(v^*_m x^*_m\) in the total input:

\[
\sum_{m} v^*_{m} x^*_{m} = (1), \quad (3.3)
\]

we can derive the relative importance of each item with reference to the value of each \(v^*_m x^*_m\). The same holds for \(u^*_s y^*_s\), where \(u^*_s\) provides a measure of the relative contribution of \(y^*_s\) to the overall value of \(\theta^*\). These values do not only show which items contribute to the performance of DMUo but also to what extent they do so. In other words, it is possible to express the distance frictions (or alternatively, the potential increases) in improvement projections.

In this study we use the optimal weights \(u^*_s\) and \(v^*_m\) from (3.1), and develop next our new efficiency improvement projection model. A visual presentation of this new approach is given in Figures 2 and 3.

![Figure 2 Illustration of DFM approach (Input- \(v^*_i, x^*_i\) space)](image-url)
In this approach a generalized distance friction is deployed to assist a DMU in improving its efficiency by a movement towards the efficiency frontier surface. The direction of efficiency improvement depends on the input/output data characteristics of the DMU. It is then appropriate to define the projection functions for the minimization of distance friction by using a Euclidean distance in weighted spaces. As mentioned, a suitable form of multidimensional projection functions serving to improve efficiency is given by a MOQP model which aims to minimize the aggregated input reduction frictions as well as the aggregated output augmentation frictions. Thus, the DFM approach can generate a new contribution to efficiency enhancement problems in decision analysis, by deploying a weighted Euclidean projection function, while it may address both input reduction and output augmentation.

Our DFM approach contains 5 stages which will now briefly be presented.

1. Solve DLPo in (2.3). Let the optimal objective value be $\theta^*$, and the obtained optimal weights $u_{i}^*$ and $v_m^*$.

2. Using $\theta^*$, solve (2.6)-(2.9), so that we obtain $s^{-s^*}$, $s^{++}$. Each DMU can then be categorized by $\theta^*$, $s^{-s^*}$ and $s^{++}$ as follows:

   (a) if $\theta^* = 1$, $s^{-s^*}$, $s^{++} = 0$: a situation of an efficient DMU.

   (b) if $\theta^* = 1$, $s^{-s^*} \neq 0$ or $s^{++} = 0$: improvement solutions are generated by formulas (2.13) and (2.14).

   (c) if $\theta^* \neq 1$, $s^{-s^*} \neq 0$ or $s^{++} \neq 0$: improvement solutions are generated by subsequent steps 3, 4 and 5.

3. Introduce the distance friction function $F'x$ and $F'y$ by means of (3.4) and (3.5) which are defined by the Euclidean distance shown in Figures 2 and 3. And solve the following MOQP using $d^x_{m}$ (a reduction distance for $x_m$) and $d^y_{so}$...
(an augmentation distance for $y_{so}$) as variables:

\[
\min \ F_{r^x} = \sqrt{\sum_m (v^*_m x_{mo} - v^*_m d^*_m)^2} \tag{3.4}
\]

\[
\min \ F_{r^y} = \sqrt{\sum_s (u^*_s y_{so} - u^*_s d^*_s)^2} \tag{3.5}
\]

s.t. \[
\sum_m v^*_m (x_{mo} - d^*_m) = \frac{2\theta^*}{1 + \theta^*} \tag{3.6}
\]

\[
\sum_s u^*_s (y_{so} + d^*_s) = \frac{2\theta^*}{1 + \theta^*} \tag{3.7}
\]

\[
x_{mo} - d^*_m \geq 0 \tag{3.8}
\]

\[
d^*_m \geq 0 \tag{3.9}
\]

\[
d^*_s \geq 0 \tag{3.10}
\]

where $x_{mo}$ is the amount of input item $m$ for an arbitrary inefficient DMUo, and $y_{so}$ is the amount of output item $s$ for arbitrary inefficient DMUo.

The aim of function $F_{r^x}$ (3.4) is to find a solution that minimizes the sum of input reduction distances which is incorporated in the improvement friction. The aim of function $F_{r^y}$ (3.5) is to find a solution that minimizes the sum of output augmentation distances which is incorporated in the improvement friction.

Constraint functions (3.6) and (3.7) refer to the target values of input reduction and output augmentation. An illustration of a target value and a ‘fair’ allocation between input efforts and output efforts is shown in Figure 4.

The fairness in the distribution of contributions from the input and output side to achieve efficiency is established as follows. The total efficiency gap to be covered by inputs and outputs is $(1-\theta^*)$. The input and output side contribute according to their initial levels $\theta$ and $\theta^*$, implying shares $\theta^*/(1+\theta^*)$ and $1/(1+\theta^*)$ in the improvement contribution. Hence the contributions from both sides equal $(1-\theta^*)[\theta^*/(1+\theta^*)]$, and $(1-\theta^*)[1/(1+\theta^*)]$.

Hence we find for the input reduction target and the output augmentation targets:

Input reduction target: \[
\sum_m v^*_m (x_{mo} - d^*_m) = 1 - (1-\theta^*) \times \frac{1}{(1+\theta^*)} = \frac{2\theta^*}{1 + \theta^*} \tag{3.11}
\]

Output augmentation target: \[
\sum_s u^*_s (y_{so} + d^*_s) = \theta^* + (1-\theta^*) \times \frac{\theta^*}{(1+\theta^*)} = \frac{2\theta^*}{1 + \theta^*} \tag{3.12}
\]
Constraint function (3.8) refers to a limitation of input reduction, while constraint functions (3.9) and (3.10) express simultaneously the pressure of input reduction and output augmentation. It is now possible to determine each optimal distance $d_{mo}^{x*}$ and $d_{so}^{y*}$ by using MOQP (3.4)-(3.10).

4. The friction minimization solution for an inefficient DMU$_o$ can now be expressed by means of formulas (3.13) and (3.14):

$$x_{mo}^{*} = x_{mo} - d_{mo}^{x*} \tag{3.13}$$

$$y_{so}^{*} = y_{so} + d_{so}^{y*} \tag{3.14}$$

5. In order to ascertain the presence of slacks for input and output variables, we have to solve formula (2.3) and (2.6)-(2.9); by using $x_{mo}^{*}, y_{so}^{*}$, we can obtain $\theta^{**}, s^{***}, s^{***}$. In this case, we are sure that $\theta^{**}$ is calculated as 1. An optimal solution for an inefficient DMU$_o$ can be now expressed by means of formulas (3.15) and (3.16):

$$x_{mo}^{**} = x_{mo}^{*} - s^{***} \tag{4.15}$$

$$y_{so}^{**} = y_{so}^{*} + s^{***} \tag{4.16}$$

By means of the DFM model, it is possible to present a new efficiency improvement solution based on the standard CCR projection. This means an increase in options for efficiency improvement solutions in DEA. The main advantage of the DFM model is that it yields an outcome on the efficient frontier that is as close as possible to the DMU’s input and output profile (see Figure 5).

In addition, the DFM model retains the property of the standard DEA approach that the measurement units of the
different inputs and outputs need not be identical, while the improvement projection in a DFM model does not need to incorporate a priori information.

Figure 5 Degree of improvement of a DFM-projection and a CCR-projection in weighted input space

4. A Goals Achievement Model in a DFM Approach

In our study we aim to integrate a GA model in the framework of the CCR-DFM model. The GA model specifies a Goal Improvement Rate (GIR) of the total efficiency gap \((1-\theta^*)\) in the DFM model. The value of GIR ranges from 0 to 1; for example, if GIR is specified to be 0.1, then the GA model can compute an input reduction value and an output augmentation value in order to achieve an improvement that is equivalent to 10% of the total efficiency gap \((1-\theta^*)\).

This model will use the constraint functions (4.1) and (4.2) instead of constraint functions (3.6) and (3.7) in the DFM model. Thus, we have the following model specification for the Goals-Achievement Values (GAVs):

\[
GAV^x = \sum_m v_m^*(x_m - d_{mo}^x) = 1 - \frac{(1-\theta^*)}{(1+\theta^*)} + \frac{(1-\theta^*)(1-GIR)}{(1+\theta^*)} = \frac{2\theta^* + (1-\theta^*)(1-GIR)}{1+\theta^*} \\
\text{(4.1)}
\]

\[
GAV^y = \sum_s u_s^*(y_s - d_{so}^y) = \theta^* + \frac{(1-\theta^*)\theta^*}{(1+\theta^*)} - \frac{(1-\theta^*)(1-GIR)\theta^*}{(1+\theta^*)} = \frac{2\theta^* - (1-\theta^*)(1-GIR)\theta^*}{1+\theta^*} \\
\text{(4.2)}
\]

A visual presentation of constraint functions (4.1) and (4.2) is given in Figure 6, which will now concisely be clarified.
Firstly, the GA model has arbitrarily specified a GIR of the total efficiency gap equal to $(1-\theta^*)$. Next, the $GAV^x$ and the $GAV^y$, which are fairly allocated between input efforts and output efforts, are computed in Figure 6, using constraint functions (4.1) and (4.2). Finally, we can compute an input reduction value and an output augmentation value in order to achieve a $GAV^x$ and a $GAV^y$ using our CCR-DFM model. If GIR = 1.0, then constraint functions (4.1) and (4.2) completely accord with constraint functions (3.6) and (3.7). In other words, the case of GIR = 1.0 represents a full improvement in the total efficiency gap $(1-\theta^*)$. Alternatively, a case GIR = 0.0 indicates a negligible improvement in the total efficiency gap $(1-\theta^*)$.

5. Application to Local Government Finance Efficiency by Means of the CCR-DFM-GA Model

5.1 Analysis framework and database of local government finance efficiency in Hokkaido, Japan

In our empirical work, we use input and output data for a set of 35 cities in Hokkaido prefecture in Japan. The cities (DMUs) used in our analysis are listed in Table 1. These cities were chosen on the basis of their population size, with a limit of 50,000, in order to avoid biased effects of scale differences in government finance.
Table 1 DMUs (Hokkaido prefecture’s cities)

<table>
<thead>
<tr>
<th>No.</th>
<th>DMU</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sapporo</td>
<td>1,880,863</td>
</tr>
<tr>
<td>2</td>
<td>Asahikawa</td>
<td>355,004</td>
</tr>
<tr>
<td>3</td>
<td>Hakodate</td>
<td>294,264</td>
</tr>
<tr>
<td>4</td>
<td>Kushiro</td>
<td>190,478</td>
</tr>
<tr>
<td>5</td>
<td>Tomakomai</td>
<td>172,758</td>
</tr>
<tr>
<td>6</td>
<td>Obihiro</td>
<td>170,580</td>
</tr>
<tr>
<td>7</td>
<td>Otari</td>
<td>142,161</td>
</tr>
<tr>
<td>8</td>
<td>Kitami</td>
<td>129,365</td>
</tr>
<tr>
<td>9</td>
<td>Ebetsu</td>
<td>125,601</td>
</tr>
<tr>
<td>10</td>
<td>Muroran</td>
<td>98,372</td>
</tr>
<tr>
<td>11</td>
<td>Iwamizawa</td>
<td>93,677</td>
</tr>
<tr>
<td>12</td>
<td>Chitose</td>
<td>91,437</td>
</tr>
<tr>
<td>13</td>
<td>Eniwa</td>
<td>67,614</td>
</tr>
<tr>
<td>14</td>
<td>Kitahiroshima</td>
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</tr>
<tr>
<td>15</td>
<td>Ishikari</td>
<td>60,104</td>
</tr>
<tr>
<td>16</td>
<td>Noboribetsu</td>
<td>53,135</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No.</th>
<th>DMU</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hokuto</td>
<td>48,056</td>
</tr>
<tr>
<td>2</td>
<td>Takikawa</td>
<td>45,562</td>
</tr>
<tr>
<td>3</td>
<td>Abashiri</td>
<td>42,045</td>
</tr>
<tr>
<td>4</td>
<td>Wakkanai</td>
<td>41,592</td>
</tr>
<tr>
<td>5</td>
<td>Date</td>
<td>37,066</td>
</tr>
<tr>
<td>6</td>
<td>Nayoro</td>
<td>31,628</td>
</tr>
<tr>
<td>7</td>
<td>Nemuro</td>
<td>31,202</td>
</tr>
<tr>
<td>8</td>
<td>Bibai</td>
<td>29,083</td>
</tr>
<tr>
<td>9</td>
<td>Rumoi</td>
<td>26,826</td>
</tr>
<tr>
<td>10</td>
<td>Monbetsu</td>
<td>26,632</td>
</tr>
<tr>
<td>11</td>
<td>Fukagawa</td>
<td>25,838</td>
</tr>
<tr>
<td>12</td>
<td>Furano</td>
<td>25,076</td>
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<tr>
<td>13</td>
<td>Shibetsu</td>
<td>23,411</td>
</tr>
<tr>
<td>14</td>
<td>Sunagawa</td>
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</tr>
<tr>
<td>15</td>
<td>Ashibetsu</td>
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</tr>
<tr>
<td>16</td>
<td>Akabira</td>
<td>14,401</td>
</tr>
<tr>
<td>17</td>
<td>Yubari</td>
<td>13,001</td>
</tr>
<tr>
<td>18</td>
<td>Mikasa</td>
<td>11,927</td>
</tr>
<tr>
<td>19</td>
<td>Utashinai</td>
<td>5,221</td>
</tr>
</tbody>
</table>

For our DEA, we use the following inputs and outputs:

Input:
(1) Expenditures by local government
(1) City bonds

Output:
(1) Tax revenues by local government

Data on the two input variables and the one output variable were obtained from ‘The Municipality Accounting Card 2005, Ministry of Internal Affairs and Communications, Japan’.

In our application, we first applied the standard CCR model, while next the results of this analysis are used to determine the CCR-DFM and CCR-DFM-GA projections. The steps followed in our analysis are shown in Figure 7.

In Subsection 5.2, we will present the efficiency evaluation results based on the CCR model. Next, in Subsection 5.3, we will present the efficiency improvement projection results based on the CCR-DFM model, and compare these with the CCR projections and outcomes. Finally, in Subsection 5.4, we will present the efficiency improvement projection results based on the CCR-DFM-GA model.
5.2 Efficiency evaluation based on the CCR model

The efficiency evaluation results for the 16 larger cities (more than 50,000 population) and the smaller 19 cities (less than 50,000 population) based on the CCR model are given in Figures 8 and 9.

From Figure 8, it can be seen that Tomakomai city, Ebetsu city and Chitose city are efficiently operating cities. From these results, it is clear that Tomakomai city has a large scale industrial area and a harbor, while Chitose has a New Chitose International Airport. And Ebetsu city has brick works which is one of the best productions in Japan, while also many universities have located there. On the other hand, Iwamizawa city has a low efficiency (i.e., an efficiency score below 50%) in terms of government finance. It is also clear that this city has in the past flourished from its coal production and its railway transportation, but most coal mines in Hokkaido were closed down after 1970’s.

From Figure 9, it can be seen that Hokuto city and Takikawa city are efficient. It is noteworthy that Hokuto city has promoted mergers of cities, towns and villages, in order to improve the city administration’s efficiency. Furthermore, this city has a large scale factory as a subsidiary of a cement company in Japan. And Takikawa city has successfully developed agriculture, industry, commerce, and sightseeing activities. On the other hand, Akabira city, Yubari city and Utashinai city are low efficiency (i.e., an efficiency score below 50%) cities in terms of government finance. It is also noteworthy that these cities have flourished as a previous coal mining area, but now these cities were deprived from their main industry.
Figure 8 Efficiency score based on CCR model (16 larger cities, more than 50,000 population)

Figure 9 Efficiency score based on CCR model (19 smaller cities, less than 50,000 population)
5.3 Efficiency improvement projection based on CCR and CCR-DFM models

Efficiency improvement projection results based on the CCR and CCR-DFM model for inefficient cities are presented below (see Tables 2 and 3).

In Tables 2 and 3, it appears that the ratios of change in the CCR-DFM projection are smaller than those in the CCR projection, as was expected. Especially, Sapporo, Kushiro, Muroran and Noboribetsu in Table 1, which are non-slack type cities (i.e., $s^{-*}$ and $s^{+*}$ is zero), became marked. The CCR-DFM projection involves both input reduction and output augmentation, and clearly, the CCR-DFM projection does not involve a uniform ratio because this model looks for the optimal input reduction (i.e., the shortest distance to the frontier, or distance friction minimization). For instance, the CCR projection shows that Sapporo should reduce the local expenditure and city bonds by 13.55% to become efficient. The CCR-DFM results show that only a reduction in expenditures of 7.91% and an augmentation in the revenues of 7.27% are required to become efficient. Apart from the practicality of such a solution, the models show clearly that a different, and a perhaps more efficient solution is available than the standard CCR projection to reach the efficient frontier.

<table>
<thead>
<tr>
<th>DMU</th>
<th>Score(θ)**</th>
<th>Score(θ)**</th>
<th>Difference</th>
<th>% Difference</th>
<th>Score(θ)**</th>
<th>Score(θ)**</th>
<th>Difference</th>
<th>% Difference</th>
</tr>
</thead>
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<td>I/O</td>
<td>Data</td>
<td>I/O</td>
<td>Data</td>
<td>I/O</td>
<td>Data</td>
<td>I/O</td>
<td>Data</td>
</tr>
<tr>
<td>Sapporo</td>
<td>0.865</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expenditures</td>
<td>1,087,830,056.4</td>
<td>13.55%</td>
<td>-63,468,932.0</td>
<td>-7.91%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>City bonds</td>
<td>822,100</td>
<td>-83,492,68.6</td>
<td>-13.55%</td>
<td>0.0</td>
<td>0.00%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenues</td>
<td>261,122,978</td>
<td>0.0</td>
<td>0.00%</td>
<td>189,765,555</td>
<td>7.27%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Muroran</td>
<td>0.873</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Expenditures</td>
<td>3,333,732</td>
<td>552,431.5</td>
<td>-12.71%</td>
<td>0.0</td>
<td>0.00%</td>
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<tr>
<td>City bonds</td>
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<td>-12.71%</td>
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<td>0.00%</td>
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<tr>
<td>Revenues</td>
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<td>0.00%</td>
<td>977,165.6</td>
<td>6.79%</td>
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<td>Asahikawa</td>
<td>0.878</td>
<td>1.000</td>
<td>1.000</td>
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<td></td>
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<tr>
<td>Expenditures</td>
<td>1,494,436,059</td>
<td>32.24%</td>
<td>-287,167,015</td>
<td>19.22%</td>
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<tr>
<td>City bonds</td>
<td>1,559,180</td>
<td>-440,417.2</td>
<td>-28.13%</td>
<td>0.0</td>
<td>0.00%</td>
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<td></td>
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<tr>
<td>Revenues</td>
<td>380,679</td>
<td>0.0</td>
<td>0.00%</td>
<td>74,189,901</td>
<td>19.22%</td>
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<td></td>
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<tr>
<td>Hakodate</td>
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<td>1.000</td>
<td>1.000</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expenditures</td>
<td>1,264,649,751</td>
<td>35.42%</td>
<td>-279,050,943</td>
<td>21.52%</td>
<td></td>
<td></td>
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<tr>
<td>City bonds</td>
<td>1,346,180</td>
<td>-56,004,31</td>
<td>-34.10%</td>
<td>0.0</td>
<td>0.00%</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Revenues</td>
<td>319,186</td>
<td>0.0</td>
<td>0.00%</td>
<td>68,702,289</td>
<td>21.52%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kushiro</td>
<td>0.544</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expenditures</td>
<td>1,375,039,059</td>
<td>45.62%</td>
<td>-348,952,573</td>
<td>32.50%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>City bonds</td>
<td>1,075,870</td>
<td>-647,022.1</td>
<td>-50.49%</td>
<td>0.0</td>
<td>0.00%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenues</td>
<td>222,484,38</td>
<td>0.0</td>
<td>0.00%</td>
<td>67,500,24</td>
<td>29.55%</td>
<td></td>
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</tr>
<tr>
<td>Obihiro</td>
<td>0.746</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expenditures</td>
<td>7,373,151,206</td>
<td>25.40%</td>
<td>-106,390,972</td>
<td>14.55%</td>
<td></td>
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<tr>
<td>City bonds</td>
<td>715,030</td>
<td>-22,022,12</td>
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<td>0.00%</td>
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<tr>
<td>Revenues</td>
<td>2,079,04</td>
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<td>30,069,74</td>
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<tr>
<td>Kitami</td>
<td>0.586</td>
<td>1.000</td>
<td>1.000</td>
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</tr>
<tr>
<td>Expenditures</td>
<td>8,000,870</td>
<td>28.59%</td>
<td>-1,102,203,089</td>
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<td>333,724</td>
<td>-7,729,03</td>
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<td>0.0</td>
<td>0.00%</td>
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<tr>
<td>Revenues</td>
<td>1,428,86</td>
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<td>0.00%</td>
<td>1,915,675.7</td>
<td>13.10%</td>
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</tr>
</tbody>
</table>

Table 2 Efficiency improvement projection results of CCR and CCR-DFM model (more than 5000 population cities)
5.4 Efficiency improvement projection of the CCR-DFM-GA models

We will now provide a comprehensive picture of the results of our integrated CCR-DFM-GA model, and use Yubari city as a reference (‘target’) city. Yubari city was in a financial crisis in March 2007. Now this city has a local government that is responsible for a financial reconstruction, and hence it has put local public finance on a recovery track. But the city has not enough strength left to a complete efficiency improvement, as shown in Table 3.

In this subsection, we will use as an inefficient reference city (DMU) Yubari city, and present an efficiency improvement projection result based on the CCR-DFM-GA model. We assume that the GIR uses steps from 0.0 to 1.0 at intervals of 0.1. Next, the efficiency scores and the input reduction values and the output augmentation values based on the CCR-DFM-GA model are calculated in Table 4 and Figure 10.

These results show that if the city executes an efficiency improvement plan of a GIR amounting to 0.4 (i.e., 40% of the total efficiency gap), only a reduction in the expenditure of 29.91% and an augmentation in the tax revenues of 19.83% are required, and then the efficiency score improves from 0.337 to 0.504. And the results of a plan of GIR 1.0 (i.e., 100% of the total efficiency gap) accord with the result of our CCR-DFM model in Table 3.

These results may offer a meaningful contribution for decision making and planning of local government finance. And this new model may thus become a policy instrument that may have a great added value for decision making and planning of both public and private actors.
Table 4 Efficiency improvement projection results based on the CCR-DFM-GA model (Yubari city)

<table>
<thead>
<tr>
<th>GIR</th>
<th>Score</th>
<th>$d_x^{\text{Expenditures}}$</th>
<th>$d_x^{\text{City bonds}}$</th>
<th>$d_x^{\text{Tax revenues}}$</th>
<th>Expenditures (%)</th>
<th>City bonds (%)</th>
<th>Tax revenues (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.337</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>0.1</td>
<td>0.372</td>
<td>-943471.1</td>
<td>0.0</td>
<td>46925.2</td>
<td>-7.48%</td>
<td>0.00%</td>
<td>4.96%</td>
</tr>
<tr>
<td>0.2</td>
<td>0.411</td>
<td>-1886942.2</td>
<td>0.0</td>
<td>93850.3</td>
<td>-14.95%</td>
<td>0.00%</td>
<td>9.91%</td>
</tr>
<tr>
<td>0.3</td>
<td>0.455</td>
<td>-2830413.2</td>
<td>0.0</td>
<td>140775.5</td>
<td>-22.43%</td>
<td>0.00%</td>
<td>14.87%</td>
</tr>
<tr>
<td>0.4</td>
<td>0.504</td>
<td>-3773884.3</td>
<td>0.0</td>
<td>187700.7</td>
<td>-29.91%</td>
<td>0.00%</td>
<td>19.83%</td>
</tr>
<tr>
<td>0.5</td>
<td>0.559</td>
<td>-4717355.4</td>
<td>0.0</td>
<td>234625.8</td>
<td>-37.38%</td>
<td>0.00%</td>
<td>24.78%</td>
</tr>
<tr>
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<td>0.623</td>
<td>-5660826.5</td>
<td>0.0</td>
<td>281551.0</td>
<td>-44.86%</td>
<td>0.00%</td>
<td>29.74%</td>
</tr>
<tr>
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<td>0.696</td>
<td>-6403669.2</td>
<td>-35911.5</td>
<td>328476.2</td>
<td>-50.75%</td>
<td>-3.13%</td>
<td>34.70%</td>
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<tr>
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<td>0.780</td>
<td>-6875404.7</td>
<td>-120350.0</td>
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<td>-54.49%</td>
<td>-10.48%</td>
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<td>44.61%</td>
</tr>
<tr>
<td>1.0</td>
<td>1.000</td>
<td>-7818875.6</td>
<td>-289226.8</td>
<td>469251.7</td>
<td>-61.96%</td>
<td>-25.19%</td>
<td>49.57%</td>
</tr>
</tbody>
</table>

Figure 10 Efficiency improvement projection results based on the CCR-DFM-GA model (Yubari city)
6. Conclusion

In this paper we offer a new methodology for an inefficient city to reach the efficient frontier and to achieve the prior goal. This methodology does not require a uniform reduction in all inputs, as in the standard model. Instead, the new method minimizes the distance friction for each input and output. As a result, the reductions in inputs and augmentations in outputs necessary to reach the efficient frontier are smaller than in the standard model. Furthermore, our CCR-DFM-GA model can present a more realistic efficiency improvement plan, and thus may provide a meaningful contribution to decision making and planning for efficiency improvement of relevant agents.

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